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Verification of Crystal Elastic Anisotropy Theory by Ultrasonic Diffraction Experiments

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This paper points out an indirect but convincing experimental verification of certain theories concerning elastic-wave propagation in anisotropic media. Relations between the Poynting vector and the propagation vector for longitudinal wave propagation near, but not exactly parallel to, certain pure-mode axes permit the calculation of the ultrasonic diffraction from large single apertures (transducers) oriented for propagation exactly along the pure-mode axes. Then the diffraction loss *versus* distance, measured in oriented single crystals, permits the verification of the theories concerning these relations and concerning ultrasonic diffraction. It is shown that five items of theory have been verified by the ultrasonic diffraction measurements.

Introduction

Previous theoretical work by Musgrave (1954) and Waterman (1959) has demonstrated that for propagation directions near pure-mode axes of three, four, and sixfold symmetry, the phase velocity of longitudinal ultrasonic waves can be expressed as $v=v_0(1-b\theta^2)$, neglecting terms of higher order in θ. The angle θ is measured between the pure-mode axis and the propagation vector $\boldsymbol{\beta}$. The coefficient b is a function of the elastic moduli of the crystal, a different function for each pure-mode axis. The velocity along the pure-mode axis is v_0. Since the magnitude of the propagation vector is $|\boldsymbol{\beta}|=2\pi f/v$, this magnitude can be written (Papadakis, 1963) as $|\beta|=\beta_0(1+b\theta^2)$. Lighthill (1960) has shown that the energy flow in a wave must go along the Poynting vector $\mathbf{P}$, and that the Poynting vector deviates from the propagation vector as long as the propagation vector is not parallel to a pure-mode axis in an anisotropic crystal. Waterman (1959) has written this deviation angle d_p in terms of the elastic moduli of crystals and the angle θ. It works out to be $d_p=2b\theta$ for the symmetries mentioned above. There is no azimuthal dependence for these symmetries. Since b may be positive or negative, the Poynting vector may deviate further or less far from the pure-mode axis than does the propagation vector.

Summary of theory and computations

The author utilized these theoretical concepts (Papadakis, 1963, 1964, 1966) to generalize the ultrasonic diffraction theory of Seki, Granato & Truell (1956) to include longitudinal waves along pure-mode axes of three, four, and sixfold symmetry. This theory is for circular piston sources of ultrasound and circular, coaxial, equal-sized receivers which are sensitive to both pressure and phase. Both source and receiver are many wavelengths in diameter. The theory applies to pulse-echo experiments (Roderick & Truell, 1952) with piezoelectric plate transducers on specimens with plane-parallel faces. The output of the receiver is calculated in terms of the integrated amplitude to be observed (translated into dB loss) and the apparent phase shift relative to a plane wave. It is essentially an antenna problem with every element of the transmitter radiating to every element of the receiver. The amplitude (loss) and phase are calculated as functions of distance between the planes of the transmitter and receiver. The Rayleigh integral (Seki *et al.*, 1956; Strutt, 1945) is used to express the radiation at a field point. In the generalization (Papadakis, 1963, 1964, 1966), the plane of the circular piston source is assumed exactly perpendicular to the pure-mode axis. Radiation must reach a field point by means of a Poynting vector along a radius vector $\mathbf{r}$ from an element of area on the transmitter. This Poynting vector must deviate from the pure-mode axis for most elements, and hence the propagation vector must deviate from both.

The pure-mode axis, the propagation vector, and the Poynting vector are coplanar in the case of the symmetries mentioned, however. Use of θ and d_p to express the spatial part of the phase $\boldsymbol{\beta}.\mathbf{r}$ in the Rayleigh integral results in a scalar quantity $\beta_0|\mathbf{r}|[1+b(1-2b)\theta^2]$ in the argument of the exponential function. Numerical integration over the transmitter area yields the field at a point, and subsequent integration over the receiver area yields the receiver output.

The results (Papadakis, 1966) of the numerical integrations are as follows.

(1) The loss as a function of distance is not monotonic but shows peaks and dips in the near field.

(2) The last peak, peak 'A', in loss is at $S_A=1{\cdot}6$ for $b=0$ as found by Seki *et al.* (1956). S is the Seki parameter, a normalized dimensionless distance parameter, $S=z\lambda/a^2$. Here z is distance (transmitter to receiver), λ is wavelength, and a is transmitter and receiver radius. The scale factor is the Fresnel length.

(3) The abscissa is scaled as a function of b in such a way that $S_A=0{\cdot}8(0{\cdot}5-b)$. For physical reasons, b can-

not be greater than 0·5, but may range over $-\infty < b < 0{\cdot}5$.

(4) The loss in the far field becomes monotonic increasing logarithmically. Recently, Benson & Kiyohara (1974) have shown that the rate for amplitude is indeed 6 dB per doubling of distance for $b=0$. That is, in the isotropic case the diffraction reduces to spherical spreading far from the source.

(5) The phase has plateaus where the loss has peaks. The excess phase relative to a plane wave increases asymptotically to $\pi/2$ rad.

Summary of experimental results

Pulse-echo ultrasonic attenuation experiments (Papadakis, 1960, 1963, 1964; Roderick & Truell, 1952) and velocity experiments (Papadakis, 1967) have been used to show that the theory as described predicts the diffraction behavior of ultrasonic waves in solids. In particular, the last loss peak '*A*' falls at the location S_A predicted in point (3) above. The tabulation from Papadakis (1966) is summarized in Table 1, which shows marked agreement between theory and experiment concerning the abscissa of peak '*A*'.

Table 1. *Experiments on peak positions with curcular all-plated longitudinal transducers*

Material	b	S_A theory	S_A experiment
Zn *c* axis	−5·23	0·137	not seen
Cd *c* axis	−1·408	0·410	0·6
Ge [100]	−0·581	0·74	0·7
$CaCO_3$ threefold	−0·567	0·75	0·8
Si [100]	−0·461	0·83	0·9
Quartz threefold	−0·250	1·07	1·1
NaCl [111]	−0·212	1·12	1·3
Steel	0·000	1·60	1·8
Si [111]	0·162	2·37	2·4
NaCl [100]	0·196	2·63	2·7
KBr [100]	0·373	6·30	6·3
KI [100]	0·380	6·67	5·7

In the velocity experiments (Papadakis, 1967), the apparent velocities calculated from raw data on travel time showed a trend *versus* total path distance. Measurements of travel time had been made between echoes 1 and N with N from 2 to 15 in various specimens. The parameters were such that the echoes were in the near and/or intermediate field of the transducers. Specimens were fused quartz ($b=0$), silicon [100] ($b=-0{\cdot}461$), silicon [111] ($b=0{\cdot}162$), and silicon [110] (b taken as 0 since it varies from positive to negative with azimuth). Subsequently, the travel times were corrected for diffraction by applying the phase shift calculated from theory (Papadakis, 1966). Recalculation of the velocities showed that the trend *versus* distance had been removed, and that the average velocity for each specimen had a smaller standard deviation than with the raw data (Papadakis, 1967).

Implications: theories verified

The attenuation and velocity data mean that the behavior (in amplitude and phase) of the waves in the near field and in the intermediate range before the far field in solids is predicted correctly by the theory. The implications are that (1) the assumptions about the piston-source nature of the transducer as transmitter and the phase-and-amplitude sensitivity of the transducer as receiver are correct, (2) the theory of Lighthill (1960) concerning the transmission of energy from point to point by the Poynting vector as if a plane wave with a definite propagation vector were involved is correct, (3) the formulation of Waterman (1959) is correct for v and d_p in terms of b and θ for longitudinal elastic waves along the three, four, and sixfold axes of symmetry, (4) the use of Waterman's (1959) formulation for v and d_p by the author (Papadakis, 1963, 1964, 1966) to derive an expression for $\boldsymbol{\beta} . \mathbf{r}$ in the Rayleigh integral (Seki *et al.*, 1956; Strutt, 1945) is correct and (5) the extension of the Rayleigh integral, first derived for fluids, to solids is correct.

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